

Q - To find an expression of electric potential energy of a charged capacitor.

Ans - Electric potential energy of a charged capacitor is equal to minimum work done by external agent in raising total charge on the plates of capacitor.

$$U = W_{\text{ext}} \quad \text{--- (i)}$$

Suppose a capacitor of capacitance C , total charge given to the capacitor is q and potential energy is U .

In the process of charging when charge raised is q' electric potential

$$V' = \frac{q'}{C}$$

Work done in increasing charge from q' to $(q' + dq')$

$$\begin{aligned} dW &= V' dq' \\ &= \frac{q'}{C} dq' \end{aligned}$$

Minimum work done by external agent in raising total charge q

$$\begin{aligned} W_{\text{ext}} &= \int dW = \int_0^q \frac{q'}{C} dq' \\ &= \frac{1}{C} \int_0^q q' dq' \\ &= \frac{1}{C} \left[\frac{q'^2}{2} \right]_0^q \\ &= \frac{1}{2C} [q^2 - 0^2] \end{aligned}$$

$$W_{\text{ext}} = \frac{q^2}{2C}$$

So electric potential energy of charged capacitor

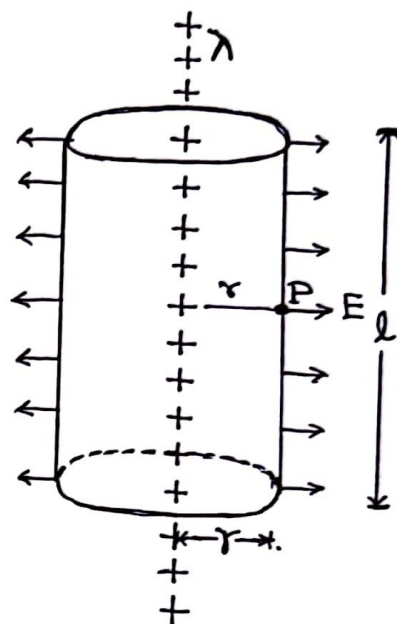
$$\left. \begin{aligned} U &= \frac{q^2}{2C} \\ &= \frac{1}{2} qV \\ &= \frac{1}{2} CV^2 \end{aligned} \right\} \text{As } q = CV$$

Q - To use Gauss law and find an expression of electric field of a long and straight line distribution of charge or thin charged conductor.

Ans - Suppose a long and straight line distribution of charge of linear density of charge λ . P is a point at perpendicular distance r .

To find expression of electric field at point P imagine a closed coaxial cylindrical surface passing through point P.

According to the condition electric field at all the points on curve surface are equal in magnitude and perpendicular to the surface, so flux of electric field through curve surface



$$\Phi_{\text{curve}} = E (\Delta S) = E (2\pi r l)$$

As electric field at all the points of two ~~curve~~ plane surfaces are parallel to the surface, so flux through plane surfaces

$$\Phi_{\text{plane}} = 0$$

Total electric flux through closed cylindrical surface

$$\Phi = \Phi_{\text{curve}} + \Phi_{\text{plane}}$$

$$\Phi = E(2\pi r l) + 0 = E(2\pi r l) \quad \text{--- (i)}$$

According to Gauss law

$$\Phi = \oint \vec{E} \cdot d\vec{l} = \frac{1}{\epsilon_0} (q_{\text{in}})_{\text{total}} = \frac{1}{\epsilon_0} (\lambda l) \quad \text{--- (ii)}$$

From relation (i) and relation (ii)

$$E(2\pi r l) = \frac{1}{\epsilon_0} (\lambda l)$$

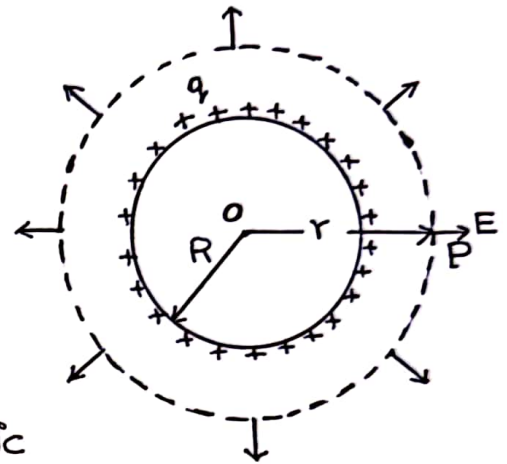
$$\therefore \boxed{E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2\lambda}{4\pi\epsilon_0 r}} \quad E \propto \frac{1}{r}$$

Q- To use Gauss law and find an expression of electric field of a charged spherical conductor.

Ans- Suppose a spherical conductor of radius R , charge q is uniformly distributed on its surface.

P is a point at distance r from the centre.

To use Gauss law and find electric field at point P , we imagine a closed concentric spherical surface passing through point P .



According to situation electric field at every point of Gaussian surface are equal in magnitude and perpendicular to surface, so electric flux through the Gaussian surface

$$\phi = E \Delta S = E(4\pi r^2) \text{ ——— (i)}$$

According to Gauss law

$$\phi = \frac{1}{\epsilon_0} (q_{in})_{total} \text{ ——— (ii)}$$

From relation (i) and relation(ii)

$$E(4\pi r^2) = \frac{1}{\epsilon_0} (q_{in})$$

$$\therefore E = \frac{1}{4\pi\epsilon_0} \frac{q_{in}}{r^2} \text{ ——— (A)}$$

Special Cases

(1) At external point ($r > R$)

$$q_{in} = q = 4\pi R^2 \sigma$$

Where σ = surface density of charge.

$$\therefore E_{ext} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{\sigma R^2}{\epsilon_0 r^2}$$

(2) At surface ($r = R$)

$$E_{\text{surface}} = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} = \frac{\sigma}{\epsilon_0}$$

(3) At internal point ($r < R$)

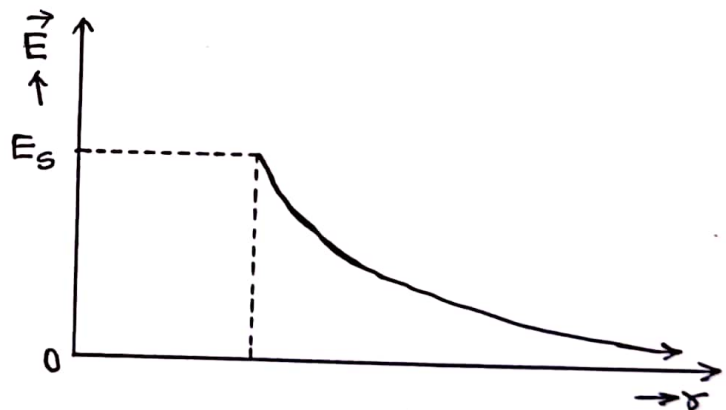
$$q_{\text{in}} = 0$$

$$\therefore E_{\text{in}} = 0$$

(4) At the centre ($r = 0$)

$$E_{\text{centre}} = 0$$

Graph of electric field



Field is

0 at internal points

Maximum at centre

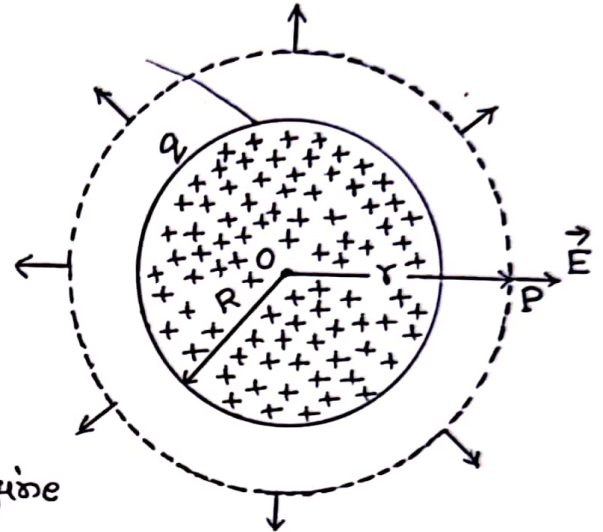
At external points

$$E_{\text{ext}} \propto \frac{1}{r^2}$$

Q- To use Gauss law and to find expression of electric field of a sphere of charge at different points.

Ans -

Suppose a sphere of charge of radius R . Charge q is uniformly distributed in entire volume. P is a point at distance r from the centre.



To find electric field by using Gauss law, we imagine a concentric spherical surface passing through point P .

According to situation electric field at all the points of Gaussian surface are equal in magnitude and perpendicular to the surface. So electric flux through spherical Gaussian surface

$$\phi = E \Delta S = E (4\pi r^2) \quad \text{--- (i)}$$

According to Gauss law

$$\phi = \frac{1}{\epsilon_0} (q_{in})_{total} \quad \text{--- (ii)}$$

From relation (i) and relation

$$E (4\pi r^2) = \frac{1}{\epsilon_0} (q_{in})_{total}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q_{in}}{r^2} \quad \text{--- (A)}$$

Case (1) - When $r > R$ (at external point)

$$q_{in} = q = \frac{4}{3}\pi R^3 \rho, \text{ where } \rho = \text{volume charge density}$$

$$\text{So } E_{ext} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{\rho R^3}{3\epsilon_0 r^2}$$

Case (2) - When $r = R$ (at surface)

$$E_{surface} = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} = \frac{\rho R}{3\epsilon_0}$$

Case(3) - When $r < R$ (at internal point)

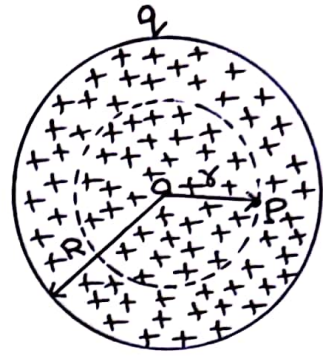
$$q_{in} = \frac{q}{(\frac{4}{3}\pi R^3)} (\frac{4}{3}\pi r^3)$$

$$q_{in} = \frac{q r^3}{R^3} = \frac{4}{3}\pi r^3 \rho$$

So

$$E_{in} = \frac{1}{4\pi\epsilon_0} \frac{q r^3}{R^3 r^2}$$

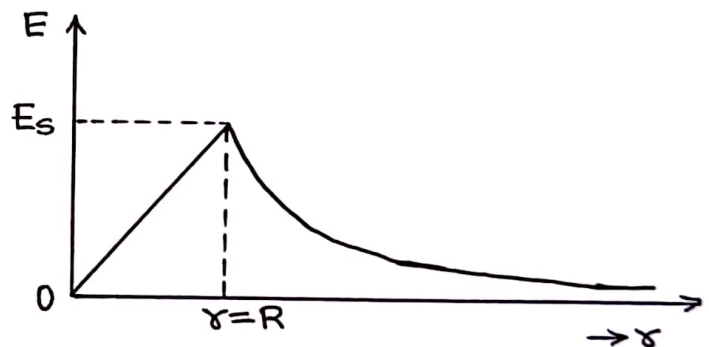
$$E_{in} = \frac{1}{4\pi\epsilon_0} \frac{q r}{R^3} = \left(\frac{\rho}{3\epsilon_0}\right) r$$



Case(4) - When $r = 0$ (at centre)

$$E_{centre} = 0$$

Graph of electric field



⇒ Field is

0 at the centre

Maximum at surface

⇒ At internal points

$$E_{in} \propto r$$

⇒ At external points

$$E_{ext} \propto \frac{1}{r^2}$$